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ENHANCING EARLY MATHEMATICAL LEARNING THROUGH MULTIMEDIA TECHNOLOGIES: A GEOGEBRA-BASED APPROACH IN PRESCHOOL EDUCATION

Multimedia technologies, encompassing images, animations, and sounds, offer innovative possibilities for preschool educators, particularly in the context of presenting mathematical content and fostering children's mathematical cognition. This study is driven by a dual objective: firstly, to formulate a GeoGebra-based model for the assimilation of fundamental mathematical concepts related to numbers and sets; secondly, to execute this model in practical educational settings. The research conducted for this paper spanned five years (2018-2023) as part of a comprehensive five-year project, comprising three distinct phases, each featuring specialized workshops. A cohort of 380 preschool teachers completed GeoGebra training, which was followed by the integration of the proposed model into the educational curriculum across preschool institutions in two districts within the Republic of Serbia. Preschool educators independently implemented two activities aligned with their project themes. The study's findings emphasize the effectiveness of implementing an interactive project-based learning approach in preschool settings, where well-integrated GeoGebra applets, particularly when harmonized with activities involving physical and tangible elements, can yield positive outcomes.

Keywords: Project-based learning; GeoGebra; preschool teachers; concepts of numbers and sets; workshops

1. INTRODUCTION

Given that the acceptance and proper understanding of mathematical concepts at preschool age builds a strong foundation for later success in mathematics and predicts school achievement in the last years (McCray, Chen 2012), there is a strong rationale for introducing preschool mathematical programs that exceed the mere practice of counting and adding. That means preschool teachers should not underestimate young children's competencies and should promote teaching concrete mathematical operations, including numerical reasoning and geometry lessons (Stipek 2013). More specifically, Brendefur et al. (2013) argue that an effective preschool curriculum demands a simultaneous focus on geometry and arithmetic processes and that geometry must be placed at an equal level of importance alongside number concepts.

Addressing the possibilities for improving geometry and arithmetic education among young children, Weiss et al. (2006) indicate that multimedia learning provides active engagement with multi presentations and enables that child to learn more deeply than from traditional modes of communication involving words and printed materials. Namely, since mobile technologies have become an integral part of their lives, preschool children already possess a certain level of digital literacy and prefer online learning materials and platforms (Reis, Ozdemir 2014). While computers offer new opportunities for learning and acquiring skills, there are concerns about potential adverse effects. These include the erosion of skills associated with traditional technologies, a negative impact on social life resulting in reduced attention, focus, learning, and memory, as well as an increased risk of gaming addiction (Tsitouridou, Vryzas 2003; Liu et al. 2021). Research conducted in Ankara, Turkey, on a sample of 111 preschool teachers has shown that educators recognized the advantages of incorporating computers into the learning experiences of young children but expressed apprehension about the potential negative impact of computers on children's well-being (Bayhan et al. 2002). Khasawneh and Al-Awidi (2008), stressed that their findings unveiled both positive and negative changes that have emerged from the utilization of computers. Concerning adverse impacts, their study demonstrated alterations in children's behavior due to computer use. More precisely, children exhibited reduced physical activity as they spent more time on the computer, leading to a decrease in time dedicated to exercise and play. Also, media outlets often feature negative reports about the detrimental impacts of digital technology on children. These reports can instill concern, anxiety, and fear in parents who worry that exposing their children to computers in preschool facilities may lead to engagement in aggres-

sive and violent computer games, rather than fostering learning, creative play, and social interaction with their peers (Mikelić Preradović et al. 2016).

Generally, there is a lack of attention and action when it comes to addressing inquiries about the organized integration of multimedia literacy into preschool education. This is because the role of Information and Communication Technology in early childhood education is still a controversial topic (Mikelić Preradović et al. 2014).

To maximize the benefits of technology for children, it is crucial for the software to be developmentally appropriate. Thoughtfully designed software should involve children in an exploratory process, sustain their interest over time, and promote active participation rather than passive observation (Judge et al. 2004).

Although various types of mathematics software have been developed to aid teaching and learning, including GeoGebra, Geometer's Sketchpad, Maple and Mathematica (Arbain, Shukor 2015; Takaci et al. 2006), for this research, we select GeoGebra. This free, open-source dynamic software is designed to combine features of a dynamic geometry software (e.g., Cabri Geometry, Geometer's Sketchpad) with the versatile possibilities of computer algebra systems (e.g., Derive, Maple) to create an integrated and easy-to-use package for teaching and learning mathematics (Saha et al. 2010). In that sense, Geogebra encourages users to approach mathematics in an experimental way (Hohenwarter, Fuchs 2005), meaning that at one level, it can be used as a dynamic geometry system while the algebra part of GeoGebra provides an insight into the relationship between the geometric aspects and their algebraic representations (Edwards, Jones 2006). When using GeoGebra, children intrigued by technology can explore geometric shapes and many mathematical ideas, thus en-joying and developing an appreciation for mathematics. At the same time, preschool teachers can create graphic representations of various mathematical concepts to engage children and facilitate their under-standing of mathematical concepts (Furner, Marinas 2014). In addition to offering dynamically linked multiple representations for versatile mathematical objects through its graphic, algebraic, and spreadsheet views (Hohenwarter, Lavicza 2009), GeoGebra provides an excellent opportunity for creating an online learning environment through sharing customized interactive worksheets on GeoGebraWiki (Hohenwarter, Lavicza 2007).

The role of GeoGebra in mathematics education from the primary to university level has been investigated in numerous studies (Vernadakis et al. 2005; Carter, Ferrucci 2009; Bakara et al. 2010; Hall, Chamblee 2013; Arbain, Shukor 2015; Korenova 2017; Tay, Wonkyi 2018; Dockendorff, Solar 2018; Zulnaidi et al. 2020), however, this paper focuses on the preschool level and address the issue of adopting the con-

cepts of number and set in preschool institutions. Since the aim of this study involves designing a model for the use of mathematical software GeoGebra in educational work with preschool children and its implementation in teaching basic mathematical concepts of numbers and sets, the research task implied an investigation of the impact of GeoGebra activities on the acquisition of concepts of numbers and sets through an integrated learning approach. Hence, we initially postulated that applying the mathematical software GeoGebra in project-based educational work in preschool institutions in Serbia was possible.

In this light, it is noteworthy to acknowledge that volunteers from an extensive self-supporting user community have translated GeoGebra into numerous languages enabling usage in local languages and multicultural environments.

As multimedia technologies provide creative opportunities for teachers in preschool institutions by integrating images, animations, and sounds, we hypothesize that the interactive applet created in GeoGebra facilitates the acquisition of basic mathematical concepts of numbers and sets.

The second aspect that needed consideration was the problem of knowing and using communication technologies and math software by preschool teachers and the need for adequate training to increase their readiness and desire to use GeoGebra. This has led to the formulation of another hypothesis stating that adequate training increases the eagerness and desire of educators to use the GeoGebra software. Therefore, our research is also related to developing additional digital competencies of preschool teachers to integrate mathematics into everyday activities in preschool institutions.

This paper is structured as follows: after the introduction, the next section outlines the research methods employed in Serbian preschool institutions as part of a five-year project conducted from 2018 to 2023. This is followed by presenting the results and discussion in a single section. Finally, the paper concludes with a summary and implications of the research.

2. RESEARCH METHODOLOGY

Authors have devised a structured three-phase model for the effective implementation of GeoGebra mathematics software in Serbian preschool institutions.

2.1. Introduction Phase - Survey and Data Collection Phase

During the initial phase of the project, which was implemented in February 2018, a group of 97 preschool educators was selected to receive an introduction to the fundamentals of GeoGebra. These educators had varying years of work experience and were drawn from 22 different preschool institutions within three districts: South Bačka, Moravički District, and Srem District. The aim was to gather data on IT infrastructure, IT literacy, and to investigate their opinions on potential possibilities for implementing the GeoGebra application in working with preschool-aged children. Ensuring the quality of this foundational training was of paramount importance during this stage. Subsequently, a specialized open-ended questionnaire tailored explicitly for this research phase was employed to gather data. Following a favourable reception of the GeoGebra software's potential in preschool institutions, the research advanced to the subsequent phase. In this phase, introducing preschool teachers to the basics of GeoGebra software is crucial. It establishes a foundation by equipping educators with the necessary knowledge and skills to effectively utilize GeoGebra in their teaching. The quality of this initial training phase is paramount as it shapes their understanding of the software's potential for enhancing mathematics education in preschool institutions. Conducting surveys with the participating preschool teachers is vital for several reasons. It gathers valuable insights into their experience levels, which can influence their readiness to adopt GeoGebra. Additionally, it gauges their initial impressions and potential concerns, helping researchers tailor subsequent phases to address specific needs. The data collected in this phase serves as a baseline for evaluating the impact of later training and implementation.

2.2. Workshop and Training Phase

The second phase involves hands-on workshops and training sessions, where educators learn to create interactive GeoGebra applets. In this research phase, we selected two specific GeoGebra activities that align with the project-based pedagogical approach recently adopted in Serbian preschool institutions, based on the New Foundations of Preschool Education Program. Considering that Sirmium Higher School is located in the Srem District, and that our branch office is situated in Čačak (in the Moravički District), the second phase of education was completed by 380 educators from these two districts. Over a span of five years (2018-2023), we conducted workshops that provided comprehensive guidance to preschool teachers on the independent

creation of two distinct types of interactive GeoGebra applets. During the first two years, these workshops were conducted in person, while in response to the pandemic, they transitioned to freely accessible online events.

As part of their feedback, each workshop participant generated and submitted their interactive GeoGebra applet, either in the form of a GeoGebra (ggb) document or through a provided link, after establishing their account on the GeoGebra platform.

This phase is of great significance as it bridges the gap between theoretical knowledge and practical application. It empowers teachers with the skills needed to effectively integrate GeoGebra into their teaching methods. Furthermore, it fosters a community of practice among educators who can collaborate and share innovative teaching resources. During that period, and following each conducted workshop, educators working with children in the preschool preparatory group continued with the implementation of the project's third phase.

The first interactive GeoGebra applet aims to understand the concept of numbers adequately.

Preschool teachers can independently create activities in GeoGebra on a given topic while implementing any project within their educational work. Depending on the topic, an interactive GeoGebra applet with images is formed within that project, in which the number of visible images changes by clicking the mouse. This is achieved by typing two simple commands on the scripting tab (On Click) within each image:

```
SetValue[a,!a]
If[a,SetDynamicColour(image1,1,1,1,1),If[a==false,SetDynamicColour(image1,1
,1,1,0.2)]]
```

In the settings, the image number and the name of the check box are changed for each image. The number of check boxes must be the same as the number of images. The visibility of all unnecessary elements is changed, i.e., they are removed from the image.

At the same time, the numerical record of the number changes as well. Figure 1 shows the appearance of an interactive GeoGebra applet within a project on the topic 'Means of Transport'. The applet contains drawings created by children aged 5 and 6.

As a result of this project phase, a comprehensive guide on creating GeoGebra applets related to the concept of numbers has been published in the electronic textbook (Pesic, Dedaj 2018).

Figure 1. Example of an interactive GeoGebra applet on the topic 'Means of Transport' https://www.geogebra.org/m/wes4gqkr

The second designed interactive GeoGebra applet deals with the concept of a set, i.e., the series and counting of set elements with common properties. A circular line represents the set, and the set elements depend on the project theme. The inserted elements can be moved with the mouse, and the numerical recordings of the number of elements inside and outside the set can also be seen. The number of elements inside and outside the set is counted by entering the following commands in the input line:

CountIf((x(A))^(2)+(y(A))^(2)<64,A,{A,B,C,D,E,F,G,H,I})
CountIf((x(A))^(2)+(y(A))^(2)>64,A,{A,B,C,D,E,F,G,H,I})

Afterwards, the values of the obtained parameters a and b are placed on the Graphic view using the command: Text(a). Figure 2 shows an example of an interactive GeoGebra applet with the 'Geometric Shapes' theme.

The creation of the second GeoGebra applet on the topic of the concept of sets is extensively described in a relevant scientific paper (Pesic, Dedaj, Pesic 2023).

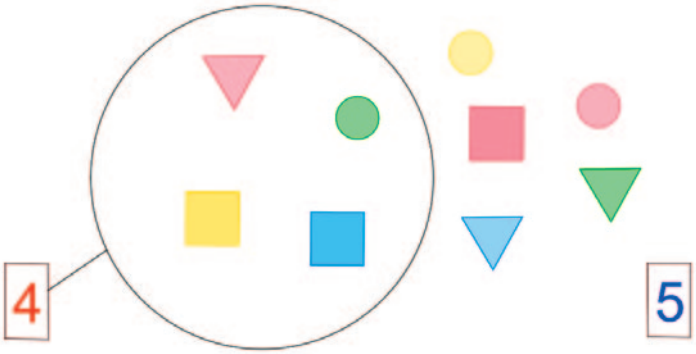

Figure 2. Example of an interactive GeoGebra applet on the topic 'Geometric Shapes' https://www.geogebra.org/m/hbdmgqck

2.3. Monitoring and Implementation Phase

The third phase of our research involved monitoring the outcomes of activities implemented in preschool institutions. During this phase, preschool teachers integrated GeoGebra activities into various project themes. These activities often included scanned drawings created by children on the given topic before interacting with the GeoGebra applets. The third phase also encompassed the monitoring of children's reactions and the collection of preschool teachers' impressions.

This phase holds a pivotal role within the broader research project. It entails a close examination and evaluation of the impact of GeoGebra-based activities introduced in preschool settings. Its significance is multifaceted, encompassing the assessment of practical implementation, the observation of child engagement, the gathering of teacher impressions, and the extensive data collection from a diverse sample.

Ultimately, this phase contributes essential insights for the advancement and implementation of technology-enhanced mathematics education in preschool institutions, shaping the trajectory of future developments in this field.

3. RESULTS AND DISCUSSION

In the initial phase of our research, which involved a survey based on an open-ended questionnaire, we engaged 97 preschool teachers working across 22 preschool institutions across three districts in the Republic of Serbia: South Backa, Moravički, and Sremski. The significance of this sample lies in its representation of a diverse cross-

section of educators from various institutions within the districts. The following discussion presents key findings and insights drawn from this survey.

The surveyed preschool teachers exhibited varying levels of professional experience (Fig. 3), with 46% having 0 to 10 years, 36% having 11 to 20 years, and 18% having over 20 years of experience.

Figure 3. Professional experience

To address the first question in the questionnaire, "What IT equipment does the institution you work in have?" the responses revealed that 95% of the surveyed institutions possess a television, 85% have access to a laptop or computer, 55% are equipped with a video projector, and 15% possess a smart board (Fig. 4). It is noteworthy that one respondent reported their institution's possession of a luminous table, while seven respondents emphasized the need for additional equipment to cater to all educational groups. These findings underscore the existing infrastructure for IT integration in preschool institutions, while also highlighting room for potential enhancement.

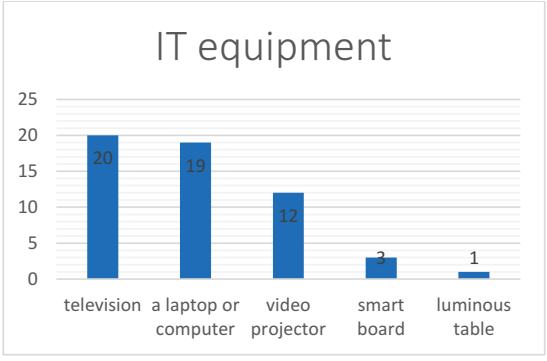

Figure 4. IT equipment

The subsequent question aimed to evaluate the IT literacy of respondents and solicit brief descriptions of their digital competencies. A significant portion of participants, 38%, self-evaluated their digital competencies as either excellent or advanced, while 48% assessed their technology skills as average, satisfactory, solid, or sufficient for their professional requirements. A mere 14% indicated a lack of proficiency in IT (Fig.5). Notably, when questioned about prior exposure to GeoGebra, a substantial 76% of respondents reported no prior encounters, indicating a potential knowledge gap that may be addressed through training and support initiatives.

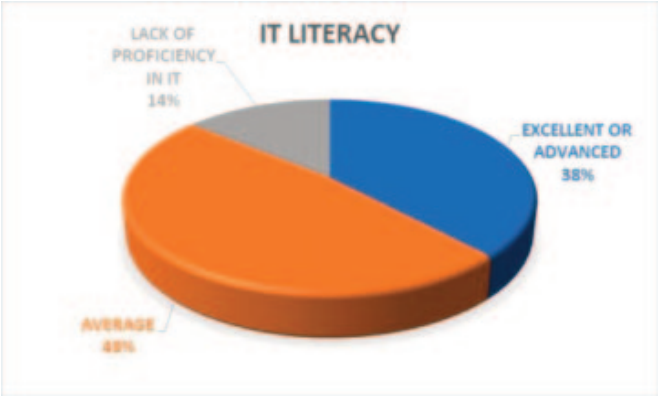

Figure 5. IT Literacy

Following this, participants were asked whether they believed there were opportunities to integrate GeoGebra into their educational practices with preschool children. Impressively, 97% of the respondents expressed a positive outlook on the feasibility of GeoGebra integration. A particularly illuminating response underscored the versatility of technology in contemporary education, with a participant highlighting the potential to scan children's drawings and incorporate them into GeoGebra to create engaging content that fosters children's explorative learning experiences.

This resoundingly positive response aligns with existing research findings, affirming the demand for incorporating digital tools in the teaching and learning of mathematics at the preschool level. Prior studies by Clements and Sarama (2007), Lieberman et al. (2009), Carlsen et al. (2016), and Oppermann et al. (2016) have consistently emphasized the need for such digital integration.

The final question of the survey solicited impressions of the introductory workshop. Impressively, 98% of respondents offered affirmative feedback, affirming their enthusiasm for implementing GeoGebra in their pedagogical practices. One respon-

dent succinctly stated, "The impressions are very positive, and I can say with pleasure that I would love to use this in my work with children. "These responses underscore the potential efficacy of GeoGebra training initiatives in nurturing educators' enthusiasm and readiness for incorporating digital mathematics tools into their teaching methodologies.

In conclusion of this phase, the findings from this survey of preschool teachers in Serbia underscore the readiness and enthusiasm among educators to embrace GeoGebra and other digital tools in their pedagogical practices. The results reflect the growing recognition of the importance of digital tools in early mathematics education, aligning with prior research in this domain.

The second phase of the research involved training programs for 380 preschool teachers for two specific workshops. Over the years, a significant increase in digital competencies and interests of pre-school teachers in this type of training has been observed. Due to the pandemic period, preschool institutions have become better equipped, and implementing such activities has been facilitated. After the completion of the training and the observation that preschool teachers had no problems using GeoGebra, GeoGebra activities were implemented in preschool institutions. More creative preschool teachers have come up with new ideas themselves. Some of these ideas are part of the six master's theses so far completed at the Vocational College of Education and Business Informatics - Sirmium, in which GeoGebra is used as a tool to facilitate the acquisition of some basic mathematical concepts such as measurement concepts, spatial orientation, basic financial concepts (difference between needs and desires), pattern recognition, as well as elements of mathematics in stories for children.

All preschool teachers who had the opportunity to implement the activities in their facilities, specifically those working with children in the preparatory preschool program, were provided with instructions on how to document everything. Each activity's checklist was compiled, and anecdotal notes and photos were collected. These indicators are relevant for assessing the development of early mathematical concepts in preschool-aged children because they provide valuable insights into their cognitive and social skills during the activities.

The checklist for the first activity on the topic of 'Number' contained seven categories:

(1) Count objects carefully;

This indicator is important as it demonstrates a child's ability to grasp the fundamental concept of numbers, fostering a strong foundation for future mathematical learning.

(2) Collaborate with another child;

That reflects their social interaction skills, showcasing their ability to work together and potentially learn from peers, enhancing their overall learning experience.

(3) Verbalize their actions;

This is crucial as it indicates their capacity to articulate their thought process, helping educators understand their reasoning and identify potential misconceptions or areas of confusion.

(4) Seek additional explanations from the educator during the game;

That is relevant because it signifies a child's curiosity and willingness to seek clarification, indicating their engagement with the learning process and the educator's role in providing necessary guidance.

(5) Solve the game silently;

That reflects the child's capacity to internalize and process mathematical concepts independently, showcasing their ability to think critically and apply their knowledge without external support, a crucial skill for mathematical problem-solving.

(6) Connect the numerical record of the number with the number of objects;

This is significant as it demonstrates the child's ability to bridge the gap between abstract numerical symbols and concrete objects, a fundamental step in understanding mathematical relationships.

(7) Show no interest in the game;

This indicator is relevant because it serves as a contrasting indicator, highlighting instances where a child may not be engaged or motivated, which can be valuable information for educators to identify potential challenges or adapt teaching strategies to foster interest and participation.

There was also a section for additional observations for each child. Including a section for additional observations enhances the depth of your study, allowing for a more nuanced understanding of each child's unique learning journey and any factors influencing their mathematical development.

The study included a total of 338 children who are enrolled in a preparatory preschool program. The summary results are given in Table 1.

Table 1. Summary results of the checklist during the activity on the topic of 'Number.'

	Yes	**No**
Pay attention when counting visible objects	320 (95%)	18 (5%)
Collaborates with other children	274 (81%)	64 (19%)
Verbalizes their actions	308 (91%)	30 (9%)
During the game, seeks additional explanations from educators	180 (53%)	158 (47%)
Solves the game silently	52 (15%)	286 (85%)
Connects the numerical notation of the number with the number of objects	294 (87%)	44 (13%)
Not interested in the game	6 (2%)	332 (98%)

The high percentage (95%) of children paying attention while counting visible objects is a positive sign. It indicates that children are actively engaged in the task, which is crucial for developing their counting skills and foundational mathematical concepts. Collaboration among children is essential for building social skills, teamwork, and effective communication. The fact that 81% of children collaborate with their peers during GeoGebra activities suggests that these activities not only promote math learning but also foster important social development. When children verbalize their actions (91%), it demonstrates their capacity to express their thought processes. This skill is vital for educators to understand their reasoning, identify misconceptions, and provide appropriate guidance. Silent problem-solving (15%) can suggest that these children are comfortable with internalizing mathematical concepts and independently applying them. The relatively high percentage (53%) of children seeking additional explanations from educators reflects their curiosity and willingness to learn. This interaction with educators can enhance the learning experience and ensure that children grasp mathematical concepts effectively. The fact that 87% of children can connect numerical notation with the number of objects is a significant achievement. It demonstrates their ability to bridge the gap between abstract mathematical symbols and concrete objects, a fundamental step in mathematical understanding. The low percentage (2%) of children showing no interest in the game is a positive outcome. It indicates that the GeoGebra activities are generally engaging and appealing to the children. However, identifying and addressing those few cases of disinterest can help ensure that all children are actively involved in the learning process.

In summary, these results emphasize the effectiveness of GeoGebra activities in promoting active engagement, collaboration, and mathematical understanding among children. They underscore the importance of early math education that combines interactive tools with social interaction to foster comprehensive learning experiences.

The checklist, designed to assess the development of early mathematical concepts related to sets in preschool-aged children, was used for the second activity centred around the topic 'Set' and included eight categories

(1) Count elements in and outside of the set with attention;

This indicator is relevant as it assesses a child's ability to focus on counting accurately, a fundamental skill in understanding sets and numbers.

(2) Collaborate with another child;

That is important because it highlights the social aspect of learning, demonstrating a child's capacity to work cooperatively, share ideas, and potentially learn from peers, which can enrich their mathematical development.

(3) Verbalize their actions;

This is crucial as it allows you to explore a child's thought processes and communication skills, providing insights into their understanding of mathematical concepts and their ability to articulate their reasoning.

(4) Ask for additional explanations from the educator during the game;

That indicates a child's willingness to seek guidance when needed, reflecting their engagement with the learning process and the educator's role in supporting their mathematical growth.

(5) Solve the game silently;

This indicator showcases a child's capacity to internalize and independently apply mathematical concepts, fostering critical thinking and problem-solving skills.

(6) Connect the numerical notation of the number with the number of set elements;

That is significant as it assesses a child's ability to link abstract numerical symbols with concrete objects, a fundamental step in grasping mathematical relationships and concepts.

(7) Select set elements based on their common characteristic;

This reflects a child's understanding of set theory by identifying and grouping elements based on shared attributes, indicating progress in more advanced mathematical thinking.

(8) Show no interest in the game;

That serves as a contrasting indicator, highlighting instances where a child may lack engagement, which can inform your research by identifying potential challenges

or areas where instructional methods may need adaptation. Including a section for extra observations for each child enhances the depth of the study, providing more detailed insights into each child's progress and behaviour during these activities, thereby contributing to a more comprehensive research study.

The research included a total of 290 children. The summarized results are given in Table 2.

The exceptionally high percentage (98%) of children who pay attention while counting elements within and outside of a set underscores their engagement in mathematical tasks. This attention to detail is fundamental for understanding set concepts and forms a solid foundation for more complex mathematical thinking.

Table 2. Summary results of the checklist during the activity on the topic of 'Set.'

	Yes	**No**
Pays attention while counting elements in and out of the set	278 (96%)	12 (4%)
Collaborates with other children	270 (93%)	20 (7%)
Verbalizes their actions	274 (94%)	16 (6%)
During the game, seeks additional explanations from educators	131 (45%)	159 (55%)
Solves the game silently	28 (10%)	262 (90%)
Connects the numerical notation of the number with the number of elements in the set	250 (86%)	40 (14%)
Chooses set elements based on their common characteristic	278 (96%)	12 (4%)
Not interested in the game	4 (1%)	286 (99%)

The majority (93%) of children collaborating with their peers during GeoGebra activities reflects the promotion of teamwork and cooperative learning. These social interactions enhance not only mathematical skills but also important social skills crucial for future learning and life. The significant percentage (94%) of children verbalizing their actions suggests that they can articulate their thought processes effectively. This skill is essential for educators to assess their understanding and provide targeted support or clarification when needed. The result showing 45% of children seeking additional explanations from educators demonstrates their curiosity and willingness to learn. It highlights the valuable role of educators in facilitating understanding and addressing questions that may arise during learning activities. While 10% of children solve the game silently, this is indicative of their ability to

think independently and work through mathematical problems internally. The fact that 86% of children can connect numerical notation with the number of elements in a set is significant. It shows their progress in understanding the abstract representation of numbers in the context of sets, a critical step in mathematical development. The high percentage (96%) of children selecting set elements based on common characteristics indicates their grasp of set concepts. This ability to identify and classify objects according to shared attributes is a key aspect of set theory. The low percentage (1%) of children showing no interest in the game suggests that the GeoGebra activities are generally engaging and stimulating. Identifying and addressing the small number of cases of disinterest can help ensure that all children remain actively engaged in the learning process.

In summary, these results highlight the effectiveness of GeoGebra activities in promoting active engagement, collaborative learning, and mathematical understanding among children, particularly in the context of set concepts. They emphasize the importance of early math education that combines interactive tools with social interaction to foster comprehensive learning experiences.

After completing the activities, one of the preschool teachers wrote:

This activity is entirely new for them, especially using computers in preschool institutions. Through visual representation and with the help and suggestions of their peers, children quickly mastered all the tasks. Children enjoyed this type of activity and work method. I plan to make a new dynamic applet on the topic 'Geometric Shapes' for them in the next period because they enthusiastically solved the offered tasks.

During the final phase of the research, the possibilities of using GeoGebra software in numerous projects were demonstrated, acquiring concepts of numbers and sets interesting for preschool children. This achieved the set task of investigating the impact of the presented activities in GeoGebra on the acquisition of basic mathematical concepts of number and set, documented in the checklist's results and anecdotal notes.

4. CONCLUSIONS

The paper presents a study to explore the possibility of using GeoGebra software in teaching and learning the concepts of numbers and sets in preschool institutions in Serbia. Preschool teachers who participated in the research recognized the importance of developing digital competencies and using multimedia tools that allow preschool children to explore and visualize basic mathematical concepts and objects, thereby

acquiring and fostering mathematical knowledge. All Preschool teachers who included GeoGebra activities in their daily practice expressed only positive impressions, confirming our general postulation that it is possible to apply GeoGebra mathematical software through a game within a project-based approach in preschool institutions.

Notably, children actively participated as partners in the proposed activities, creating the interactive GeoGebra applet and the game itself. The games were not organized to last long but always followed the themes of the project being implemented, combining computer work with physical manipulation of natural objects and inserting images of objects and drawings that children made or drew, which influenced the adoption of mathematical concepts of numbers and sets. That supported the first specific hypothesis that using an Interactive applet made in GeoGebra facilitates adopting basic mathematical concepts. Preschool teachers who completed the GeoGebra training and had the opportunity to implement GeoGebra activities in their preschool institutions practically stressed that the training enabled easy use of the GeoGebra program and enabled them to become more creative and independently realize their ideas. This is reflected in an increase in their intention to continuously use this mathematical software in educational work, confirming the second specific hypothesis of the research.

Considering that the project implementation period coincided with the initiation of the reform in preschool institutions in the Republic of Serbia based on the New Foundations of Preschool Education the primary idea was to change the existing practice of implementing mathematical activities. The project-based learning approach involves the integration of all aspects of preschool upbringing and education, and the proposed activities described in the paper can be easily aligned with any project theme. Activities exclusively focused on the development of strictly mathematical concepts, as practiced in previous models, are no longer in use. Consequently, comparing differences with the previous practice has become more challenging on one hand, while on the other hand, it was necessary to find innovative methods that could align with the new working approach and incorporate mathematical concepts.

Due to the extended duration of the implementation of the second and third phases of the project, subsequent monitoring and comparison of the obtained results were possible. The majority of children participated in both activities in GeoGebra, and the results do not show significant divergences. Additionally, during the project research, the implementation of other activities involving GeoGebra was monitored. Further research was conducted within six completed Master's theses, where GeoGebra was used in working with preschool-aged children. According to the information

received from educators, children's interest remains high when GeoGebra is integrated into activities. All obtained results support the claim that there is a positive impact of applying GeoGebra on the permanent and proper acquisition of mathematical concepts related to numbers and sets.

The research findings demonstrate that using GeoGebra motivated preschool teachers and children to make educational lessons exciting and enjoyable. Preschool children collaborated in activities based on the use of GeoGebra while preschool teachers investigated the various possibilities to reach the children better and explain mathematical concepts of numbers and sets in a relevant and meaningful way. Moreover, compared to traditional teaching practices, this mathematics software has provided an opportunity for establishing an interactive and more effective teaching and learning process.

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УНАПРЕЂИВАЊЕ РАНОГ МАТЕМАТИЧКОГ УЧЕЊА ПОМОЋУ МУЛТИМЕДИЈАЛНИХ ТЕХНОЛОГИЈА У ПРЕДШКОЛСКОМ ОБРАЗОВАЊУ: ПРИСТУП БАЗИРАН НА ГЕОГЕБРИ

Сажетак:

Мултимедијалне технологије, које обухватају слике, анимације и звукове, пружају иновативне могућности за васпитаче, посебно у контексту представљања математичког садржаја и развоја математичке способности деце. Ово истраживање има двоструки циљ: прво, формулисање модела базираног на ГеоГебри за асимилацију основних математичких појмова који се односе на бројеве и скупове; друго, примена овог модела у практичним образовним оквирима. Истраживање је обухватило период од 2018. до 2023. године у оквиру петогодишњег пројекта који се реализовао у три фазе. Група од 380 васпитача успешно је завршила обуку за ГеоГебру, након чега је следило интегрисање предложеног модела у предшколски припремни програм у предшколским институцијама из два управна округа у Републици Србији – Моравичког и Сремског. Васпитачи су самостално спровели две активности у складу са темама свог пројекта. Резултати истраживања истичу ефикасност примене интерактивног приступа учењу заснованог на пројектном приступу

у предшколским оквирима, где интегрисане ГеоГебра апликације, посебно када су повезане са активностима које укључују опипљиве и реалне елементе, дају позитивне резултате.

Кључне речи: пројектно учење; ГеоГебра; васпитачи; појам броја и скупа; радионице

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